

# Emergence of spaces and the dynamic equations of FRW universes in the $f(R)$ theory and deformed Hořava-Lifshitz theory

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## Abstract

It has been shown that Friedmann equation of FRW universe can be derived from the idea which says cosmic space is emergent as cosmic time progresses and our universe is expanding towards the state with the holographic equipartition by Padmanabhan. In this note, we give a general relationship between the horizon entropy and the number of the degrees of freedom on the surface, which can be applied to quantum gravity. We also obtain the corresponding dynamic equations by using the idea of emergence of spaces in the  $f(R)$  theory and deformed Hořava-Lifshitz(HL) theory.

## 1. Introduction

Since the discovery of black hole thermodynamics[1, 2] in the 1970s, physicists have accepted the relationship between thermodynamics with horizon and gravity theory. In 1995, Jacobson[3] argued that the Einstein equation is derived from the proportionality of entropy and horizon area together with the Clausius relation  $\delta Q = TdS$  with  $\delta Q$  and  $T$  interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon, and pointed out that the Einstein equation is an equation of state. Padmanabhan[4] notices the Einstein's equation in the spherically symmetric spacetime can have a similar form as the first law of thermodynamics of the black hole horizon.

Verlinde[5] points out that gravity is explained as an entropic force caused by changes in the information associated with the positions of material bodies. With the assumption of the entropic force, the local Unruh temperature, the

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holographic principle, and the equipartition law of energy, combining with the definition of the Komar mass, he obtains the Einstein equation, where the Komar mass plays an important role.

In the FRW model, it has been shown that the Friedmann equation which describes the dynamic of the isotropic and homogeneous universe can be derived from the first law of thermodynamics with the apparent horizon, by assuming a temperature  $T = \frac{1}{2\pi r_A}$  and the entropy  $S = \frac{A}{4}$ , where  $r_A$  and  $A$  are radius and area of the apparent horizon. Cai and Kim obtain the Friedmann equation by using entropy formula for the static spherically symmetric black hole horizon in Gauss-Bonnet gravity and in more general Lovelock gravity[6]. The similar results are also obtained in scalar-tensor gravity and  $f(R)$  gravity[7], and deformed Hořava-Lifshitz(HL) gravity[8].

It is generally accepted that gravity is described as an emergent phenomenon like fluid mechanics or elasticity in recent years. Padmanabhan[9] suggested an idea that cosmic space is emergent as cosmic time progresses. Further he argued that the difference between the number of the surface degrees of freedom and that of the bulk degrees of freedom in a region of space drives the accelerated expansion of the universe through a simple equation  $\Delta V = \Delta t(N_{sur} - N_{bulk})$ , where  $V$  is the Hubble volume in Planck units and  $t$  is the cosmic time in Planck units, derived the standard Friedmann equation of FRW universe. The Gauss-Bonnet gravity and more general Lovelock gravity are also studied[10]. And emergent perspective of gravity is further investigated[11].

In this paper, we shall propose a general relationship about degrees of freedom on the surface, which is described by the relation between the number of degrees of freedom on the surface and the entropy of horizon. The relation given by Padmanabhan is based on the thought that degrees of freedom on the surface uniformly distribute over the area. But the thought isn't valid in some theories such as quantum gravity. And his relation can be reduced by our relation in the GR theory. Therefore our relation is important and general to investigate the expansion of the universe in general gravity theories. Then we use the  $f(R)$  theory and deformed HL gravity as examples to explain why the relation given by Padmanabhan is invalid but our relation is valid and obtain the corresponding dynamic equation of FRW universe.

This paper is organized as follows. In Section 2 we review the relationship between the thermodynamics of the black hole horizon and spacetime horizon, then we give a relation about the surface number of the degrees of freedom. In Section 3 we simply recall the Padmanabhan's idea about the emergence, and show the dynamic equation of FRW universe in the  $f(R)$  theory and deformed HL gravity theory under our relation. Section 4 is for conclusions and discussions. The units are chosen with  $c = \hbar = 1$ .

## 2. Description of degrees of freedom on the surface

The equivalence principle tells us that effects of gravitational field is locally indistinguishable from the effects of accelerated motion in the flat spacetime. In fact, all notions of thermodynamics are observer dependent. A local inertial observer (local falling freely observer) will not attribute to spacetime a temperature, and a local Rindler observer (local accelerated observer) with respect to local inertial observer will attribute to spacetime a non-zero temperature. Padmanabhan has found these Rindler observers around any event will attribute not only a temperature but also an entropy to the horizon in a static, spherically symmetric spacetime, and the first law of thermodynamics is also satisfied[12]. In the black hole physics, an observer who freely falls through the horizon will not attribute a temperature to the black hole while the observer who is stationary outside the black hole horizon will attribute a temperature to the black hole. Hence the relationship between the observer at rest outside the black hole horizon and the freely falling observer is exactly the same as the relationship between an Rindler observer and an inertial observer in flat spacetime[11].

Now we consider the flat spacetime for a class of Rindler observers, the metric is  $ds^2 = -dT^2 + dX^2 + dL_\perp^2$ . Using transformation of coordinates, the metric can be expressed in the form  $ds^2 = -2\kappa l dt^2 + \frac{dl^2}{2\kappa l} + dL_\perp^2$ , where  $\kappa$  is an acceleration. The form is called the Rindler metric. The Rindler metric is similar as that of black hole in the spherically symmetric spacetime if we let  $2\kappa l \equiv f(r)$  (for the review[13]).

As we know, the number of degrees of freedom are described by the entropy in the statistical physics, and an accelerated observer attributes an entropy to the horizon of spacetime. Hence we can use

$$N_{sur} = \frac{4S_H}{k_B} \quad (1)$$

to describe the number of the degrees of freedom on the surface of the horizon, where  $S_H$  is the entropy of the horizon. Because the relation between the entropy  $S_H$  and the area of its horizon is  $S_H = \frac{k_B A}{4L_P^2}$  in the GR theory and some other theories, Eq.(1) can be turned into

$$N_{sur} = \frac{A}{L_P^2} \quad (2)$$

in those theories, where  $L_P = \sqrt{\frac{\hbar G}{c^3}}$  is the Planck length. But in some gravity theories like the quantum gravity with  $S_H \neq \frac{k_B A}{4L_P^2}$ , Eq.(2) isn't valid.

An observer very close to the event horizon, performing local experiments at length scales small compared to curvature scale, has no way of distinguishing between a Rindler coordinate system in a flat spacetime and the black hole spacetime[12]. We can reduce the entropy of a Schwarzschild's black hole to

one of accelerated observers in the flat spacetime by taking the limit of the black hole of very large mass[11]. Therefore the entropy of horizon  $S_H$  can be formally replaced by that of a black hole  $S$ . So the relation between the number of degrees of freedom on the surface and the entropy form of a black hole is

$$N_{sur} = \frac{4S}{k_B}. \quad (3)$$

### 3. Emergence of space in the $f(R)$ theory and deformed HL theory

First, let us recall Padmanabhan's work[9]. He thinks that cosmic space is emergent as cosmic time progresses. To understand it, he uses a specific version of the holographic principle. For a pure de Sitter universe with a Hubble constant  $H$ , the holographic principle is expressed in the form (called holographic equipartition)

$$N_{sur} = N_{bulk}, \quad (4)$$

where the  $N_{sur}$  is the number of degrees of freedom on the spherical surface given by Eq.(2) and  $A = \frac{4\pi}{H^2}$  is the area with Hubble radius  $H^{-1}$ . Here we would like to make a note. Hubble constant  $H$  is a fundamental constant in the cosmology, and its dimension is reciprocal of the length, hence we use  $H^{-1}$  as the radius of the expanding universe can be accepted. He takes the effective number of the bulk degrees of freedom which obey the equipartition law of energy

$$N_{bulk} = |E|/(1/2)k_B T \quad (5)$$

and the horizon temperature

$$k_B T = H/2\pi. \quad (6)$$

He takes  $|E|$  to be the Komar energy  $|(\rho + 3p)|V$  contained inside the Hubble volume

$$V = 4\pi/3H^3. \quad (7)$$

For the pure de Sitter universe  $p = -\rho$ , then one can obtain the standard result

$$H^2 = 8\pi L_p^2 \rho/3. \quad (8)$$

He thinks that the expansion of the universe is being driven towards holographic equipartition(Eq.(4)), the equation is

$$\frac{dV}{dt} = L_p^2 (N_{sur} - N_{bulk}). \quad (9)$$

Using the above definition of  $V$ ,  $N_{sur}$ ,  $T$  and  $N_{bulk}$ , one obtains the relation

$$\frac{\ddot{a}}{a} = -\frac{4\pi L_p^2}{3}(\rho + 3p). \quad (10)$$

This is the standard dynamical equation of FRW universe filled by perfect fluid with energy density  $\rho$  and pressure  $p$ . Using the continuity equation  $\dot{\rho} + 3H(\rho + p) = 0$ , one gets the standard Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi L_P^2}{3}\rho, \quad (11)$$

where  $k$  is an integration constant, which can be interpreted as the spatial curvature of FRW universe. In order to have the asymptotic holographic equipartition, he takes  $(\rho + 3p) < 0$ . This implies the existence of the dark energy. The existence of the dark energy derives our universe towards the state with the holographic equipartition. His idea about the space is emergent as time processes and our universe is expanding towards the state with the holographic equipartition is creative. However he uses the idea in which the degrees of freedom on the surface uniformly distribute over the area to obtain  $N_{sur}$  (see Eq.(2)), we think that the relationship isn't general and valid in some theories, especially in the theory of quantum gravity. Subsequently, we shall consider some examples of such gravity theories.

In the  $f(R)$  theory, we will obtain a nontrivial gravitational energy-momentum tensor due to the existence of high-order curvature, which will result in a nontrivial energy density and pressure. In the deformed HL gravity, it also has a nontrivial gravitational energy-momentum tensor due to the existence of high-order curvature, the cosmological constant, and the dark radiation. But the gravitational energy density and pressure don't appear in the GR theory. Hence it is an interesting and meaningful topic to discuss the evolution of universe under this gravitational background.

Now we consider emergence of space in the  $f(R)$  theory. We take  $|E|$  to be the Komar energy which is the correct source for gravitational acceleration[14]

$$|E| = 2 \left| \int_{\Sigma} \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) U^\mu U^\nu dV \right| = |\rho + 3p| V, \quad (12)$$

where  $T_{\mu\nu}$  is the total energy-momentum tensor which contains two parts, the energy-momentum tensor of matter  $T_{\mu\nu}^{(m)}$  and gravity  $T_{\mu\nu}^{(g)}$ . One can obtain the total energy density  $\rho$  and total pressure  $p$  through  $T_{\mu\nu}$ . Here we take the black hole entropy, its form is[15]

$$S = \frac{k_B A f_R(R)}{4G} = \frac{k_B A f_R(R)}{4L_P^2}. \quad (13)$$

Thus from the Eq.(3) we obtain the number of the surface degrees of freedom

$$N_{sur} = \frac{A f_R}{L_P^2}, \quad (14)$$

where  $f_R(R)$  denotes that  $f(R)$  derivatives with respect to  $R$ . Using Eq.(5), (6) and (7), we have

$$N_{bulk} = \frac{-16\pi^2(\rho + 3p)}{3H^4}. \quad (15)$$

According to the Eq.(9), we obtain the dynamic equation

$$\dot{H} + f_R H^2 = \frac{-4\pi L_p^2}{3}(\rho + 3p). \quad (16)$$

It is the formal dynamic equation of FRW universe from emergence of space in the  $f(R)$  theory.

Up to now, we have obtained the formal dynamic equation(16) of FRW universe by the idea of emergence of space in the  $f(R)$  theory. However, we also know the total energy-momentum tensor  $T_{\mu\nu}$  can't be determined if we don't give the specific form of the theory, because it has different energy-momentum tensors in the different gravity theories.

On a large scale of universe, we know that it is isotropic and homogenous through astronomical observation. we call it cosmological principle. Under the principle, we can obtain the FRW metric

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right), \quad (17)$$

where  $k$  is the spatial curvature constant  $k = +1, 0$  and  $-1$  corresponding to a closed, flat, and open universe. Here we point out that the cosmological principle is based on astronomical observations, therefore we take the metric(17) reasonable in order to determine  $T_{\mu\nu}$ . According to the cosmological principle, we take the form[16]

$$T_{00} = \rho(t), T_{0i} = 0, T_{ij} = a^2(t)\delta_{ij}p(t), \quad (18)$$

where  $i, j$  run over 1, 2, 3.

Now we will show how to determine the total energy-momentum tensor. In the  $f(R)$  theory, the Einstein-Hilbert(EH) action can be written as

$$S = \int d^4x \sqrt{-g} (f(R) + 2\kappa^2 L_m), \quad (19)$$

where  $\kappa^2 = 8\pi G$ . Using the variational principle  $\delta S = 0$ , we obtain

$$R_{\mu\nu}f_R(R) - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\nabla^2 f_R(R) - \nabla_\mu \nabla_\nu f_R(R) = \kappa T_{\mu\nu}^{(m)}, \quad (20)$$

where  $T_{\mu\nu}^{(m)}$  is the energy-momentum tensor of the matter. Thus we have

$$G_{\mu\nu}f_R(R) = \kappa^2 T_{\mu\nu}^{(m)} + \frac{f(R) - Rf_R(R)}{2}g_{\mu\nu} + \nabla_\mu \nabla_\nu f_R(R) - g_{\mu\nu}\nabla^2 f_R(R), \quad (21)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$  is the Einstein tensor.

We assume the matter to be the perfect fluid, then the energy-momentum tensor of the matter is

$$T_{\mu\nu}^{(m)} = (\rho_m + p_m)U_\mu U_\nu + p_m g_{\mu\nu} \quad (22)$$

with  $g_{\mu\nu}U^\mu U^\nu = -1$ . Furthermore, we define

$$T_{\mu\nu}^{(g)} = \frac{1}{f_R(R)} \left[ \frac{f(R) - Rf_R(R)}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu f_R(R) - g_{\mu\nu} \nabla^2 f_R(R) \right]. \quad (23)$$

It stands for the energy-momentum tensor of the gravity caused by higher order derivative.

Therefore we obtain

$$G_{\mu\nu} = \kappa^2 \left( \frac{1}{f_R(R)} T_{\mu\nu}^{(m)} + \frac{1}{\kappa^2} T_{\mu\nu}^{(g)} \right) \equiv \kappa^2 T_{\mu\nu}. \quad (24)$$

According to the form(18), and

$$T_{00}^{(m)} = \rho_m(t), T_{00}^{(g)} = \rho_g(t), T_{ij}^{(m)} = a^2(t) \delta_{ij} p_m(t), T_{ij}^{(g)} = a^2(t) \delta_{ij} p_g(t), \quad (25)$$

we obtain

$$\rho = \frac{1}{f_R} \left[ \rho_m + \frac{1}{\kappa^2} \left( \frac{Rf_R - f}{2} - 3Hf_{RR}\dot{R} \right) \right], \quad (26)$$

and

$$p = \frac{1}{f_R} \left[ p_m + \frac{1}{\kappa^2} \left( \frac{f - Rf_R}{2} + f_{RRR}\dot{R}^2 + f_{RR}\ddot{R} + 2Hf_{RR}\dot{R} \right) \right]. \quad (27)$$

in FRW universe, where  $\dot{R}$  denotes that  $f(R)$  derivatives with respect to  $t$ . By substituting Eq.(26) and Eq.(27) into Eq.(16), we obtain the dynamic equation of FRW universe from the idea of emergence of space. Here we shall give some remarks on this result. In these papers[17], the dynamic equation of FRW universe  $\dot{H} + H^2 = \frac{-4\pi L_p^2}{3}(\rho + 3p)$  is obtained in the  $f(R)$  theory through the Einstein's equation. However, Eq.(9) globally describes the expansion of the universe while the Einstein's equation locally describes it. Therefore, it will lead to a global correction to the area in the entropy when we use Eq.(9) to describe emergence of space in the  $f(R)$  theory. That is to say, we obtain the global description of the expansion of the universe in the  $f(R)$  theory. It has  $f_R = 1$ ,  $\rho_g = p_g = 0$  when  $f(R) = R$ , then we obtain the form

$$\dot{H} + H^2 = \frac{-4\pi L_p^2}{3}(\rho_m + 3p_m), \quad (28)$$

where  $\frac{\ddot{a}}{a} = \dot{H} + H^2$ . we can see that it is nothing but the standard dynamic equation in the GR theory.

Our conclusion is presented as follows. Eq.(16) can be reduced to Eq.(28) when  $f(R) = R$ , so we confirm Eq.(16) is the correct result in the  $f(R)$  theory. we can also see that in the paper[7], the corresponding dynamic equation can't be reproduced from the entropy formula (13) in the  $f(R)$  theory, the ansatz for the temperature on the apparent horizon and the first law of thermodynamics, so we think that our work can better describe the expansion of universe based on the  $f(R)$  gravity. Moreover, we can't use the Eq.(2) to obtain the dynamic

equation of FRW universe because the entropy  $S$  isn't proportional to the area  $A$ . Here we use the Einstein's equation to determine the total energy-momentum tensor because it can't be determined if we don't give the specific form of the theory, but we don't use the specific form of the Einstein tensor. Therefore it is more convenient to determine the dynamic equation of the FRW universe. In this process, we should pay attention to the definition of Komar mass, and have to take the total energy density and total pressure in the dynamic equation of FRW universe, because it contains not only the effects of matter, but also the effects of higher-order gravity.

In the deformed HL gravity, we also take the form of Komar energy  $|E| = |\rho + 3p|V$  as the  $f(R)$  theory, the entropy has the form[18]

$$S = \frac{k_B A}{4G} + \frac{k_B \pi}{\omega} \ln \frac{A}{4G}, \quad (29)$$

where the parameter  $\omega = 16\mu^2/\kappa^2$ . Using the above formula (5), (6) and (7), we obtain the number of degrees of freedom inside the sphere

$$N_{bulk} = \frac{-16\pi^2(\rho + 3p)}{3H^4}. \quad (30)$$

Using the relation Eq.(3), we know the number of degrees of freedom on the surface:

$$N_{sur} = \frac{A}{L_P^2} + \frac{4\pi}{\omega} \ln \frac{A}{4L_P^2}. \quad (31)$$

Then we can obtain the dynamic equation by using the Eq.(9)

$$\dot{H} + H^2 = -\frac{L_P^2 H^4}{\omega} \ln \frac{\pi}{H^2 L_P^2} - \frac{4\pi L_P^2}{3}(\rho + 3p). \quad (32)$$

Since  $\frac{\ddot{a}}{a} = \dot{H} + H^2$ , we have the equation:

$$\frac{\ddot{a}}{a} = -\frac{L_P^2 H^4}{\omega} \ln \frac{\pi}{H^2 L_P^2} - \frac{4\pi L_P^2}{3}(\rho + 3p). \quad (33)$$

From this equation, we can know the standard dynamic equation of FRW universe in the GR gravity will be recovered when the parameter  $\omega \rightarrow \infty$ . Here if we use Eq.(2) in place of Eq.(3), we can't obtain the correct result, because degrees of freedom on the surface don't uniformly distribute over the area. As the  $f(R)$  theory, the total energy-momentum tensor  $T_{\mu\nu}$  can't be determined if the form of deformed HL gravity is not given, so we will use the form of deformed HL gravity to determine it. Further, the form of entropy Eq.(29) is also determined by the explicit form of deformed HL gravity.

Now we recall the HL theory, this gravity has the ADM formalism[18, 19]

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (34)$$



The action of HL gravity can be written

$$S_{HL} = \int dt d^3x N \sqrt{g} (L_K + L_V + L_M). \quad (35)$$

$$L_K = \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2), \quad (36)$$

$$L_V = \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left( \frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) - \frac{\kappa^2}{2w^4} \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left( C^{ij} - \frac{\mu w^2}{2} R^{ij} \right), \quad (37)$$

where  $L_K$  is the kinetic term,  $L_V$  is the potential term, and  $L_M$  is the matter term. The extrinsic curvature and Cotten tensor are given by

$$K_{ij} = \frac{1}{2N} (\dot{g} - \nabla_i N_j - \nabla_j N_i), \quad (38)$$

$$C^{ij} = \epsilon^{ikl} \nabla_k \left( R_l^j - \frac{1}{4} R \delta_l^j \right). \quad (39)$$

In the infrared(IR) limit, the action should be reduced to the EH action of the general relativity

$$S_{EH} = \frac{1}{16\pi G} \int d^4x N \sqrt{g} (K_{ij} K^{ij} - K^2 + R - 2\Lambda) \quad (40)$$

by setting  $x^0 = ct$ ,  $\lambda = 1$ . The speed of light, the Newton's constant and the cosmological constant are respectively given by

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}}, G = \frac{\kappa^2}{32\pi c}, \Lambda = \frac{3}{2} \Lambda_W. \quad (41)$$

By introducing a soft violation term  $\mu^4 R$ , it is called as the “deformed HL gravity” [18]. The speed of light and Newton's constant in the IR limit are given by

$$c^2 = \frac{\kappa^2 \mu^4}{2}, G = \frac{\kappa^2}{32\pi c}, \lambda = 1. \quad (42)$$

Based on these, the entropy Eq.(29) can be obtain[18].

As in the paper[19, 20], we define the energy and pressure of this universe

$$\rho \equiv \rho_m + \rho_\Lambda + \rho_k + \rho_{dr}, p \equiv p_m + p_\Lambda + p_k + p_{dr}. \quad (43)$$

Where we use the energy  $\rho_m$  and pressure  $p_m$  of perfect fluid as the ones of the matter. By introducing the cosmological constant term, the curvature term and the dark radiation term[19]

$$\rho_\Lambda = -p_\Lambda = -\frac{3\kappa^2 \mu^2 \Lambda_W^2}{8(3\lambda - 1)} \quad (44)$$

$$\rho_k = -3p_k = \frac{3k}{4(3\lambda - 1)a^2} \left( \kappa^2 \mu^2 \Lambda_W - 8\mu^4 (3\lambda - 1) + \frac{8}{\kappa^2} (3\lambda - 1)^2 \right) \quad (45)$$

$$\rho_{dr} = 3p_{dr} = \frac{3\kappa^2\mu^2}{8(3\lambda-1)} \frac{k^2}{a^4} \quad (46)$$

So far, we have already obtained the complete dynamic equation of FRW universe by Eq.(33) together with (43). The dynamic equation is derived from the idea of emergence of space. The deformed HL gravity is a quantum gravity theory, hence it describes self-consistently the physical results which contains the quantum effects. Then the Eq.(33) should be seen as the effective dynamic equation of FRW universe which contain the effective quantum gravitational effects.

The above results show that if we consider the evolution of universe in the general gravitational theories, the evolution of universe can be described by Eq.(9) according to the Padmanabhan's idea. The choice of  $N_{sur}$  is important in the Eq.(9). In general, we can use the form of the entropy of the black hole to describe that of the spacetime due to their similarity near the horizon. We give the relation between the number of the surface degrees of freedom and the entropy of the horizon of the spacetime, and take the specific forms of the  $f(R)$  gravity and deforms HL gravity to determine their effective energy-momentum tensors and the entropy. Furthermore, we can also obtain the corresponding dynamical equation of FRW universe by the equation of evolution Eq.(9).

## 4. Discussion and conclusion

In this paper, our idea mainly comes from [9] the thought that the expansion of universe is due to the difference of the number of degrees of freedom on the spherical surface and the one inside the sphere. We use also  $H^{-1}$  as the radius of the expanding universe, and take  $(\rho + 3p) < 0$  in order to have the asymptotic holographic equipartition. This also implies the existence of the dark energy. We obtain the dynamic equation of FRW universe from the idea of emergence of spaces in the  $f(R)$  theory and deformed HL theory, and confirm Padmanabhan's idea by these examples. It should be emphasized that it is the relation  $N_{sur} = \frac{4S_H}{k_B}$  that describes the number of degrees of freedom on the surface of the horizon, because the number of degrees of freedom are described by the entropy in the statistical physics. Hence this relation is natural and general.

By establishing the relationship between the number of degrees of freedom on the spherical surface and the entropy of the black hole, using the equipartition law and the evolution equation(9), we have indeed obtained the dynamic equation of FRW universe in the  $f(R)$  theory and deformed HL theory. Here the Komar energy plays an important role which is taken as the usual form for gravitational acceleration  $|E| = 2 \left| \int_{\Sigma} (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) U^{\mu} U^{\nu} dV \right|$ . The reason why we can't simply take the energy-momentum tensor of the perfect fluid as the total energy-momentum tensor is the form of the entropy of the horizon of the spacetime and the total energy-momentum tensor are different in the different theories. Hence one has to use the specific form of gravitational theory in order to obtain the corresponding dynamical equation of FRW universe. Then,

we would also like to explain Eq.(1) again. Because of  $S = \frac{k_B A}{4L_P^2}$  in the GR theory and some gravity theories, Eq.(1) can be transformed to Eq.(2) in those theories. But in some theories such as the  $f(R)$  theory and deformed HL theory with  $S \neq \frac{k_B A}{4L_P^2}$ , one can't use Eq.(2), because degrees of freedom on the surface don't uniformly distribute over the area. Therefore our relation(Eq.(1)) is more general than Eq.(2), especially in quantum gravity theory it can be widely applied. Our results are useful for further understanding of the expansion of the universe in general gravity theories.

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